Quasisteady Modeling of Periodic Turbulent Pipe Flows

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Abstract

THE implications of using a steady-flow turbulence closure model in a quasisteady manner for the prediction of fully developed periodic flow in a long circular pipe are examined. Using the results of a simple one-equation model and the data from recent experiments, the usefulness and limitations of quasisteady modeling are discussed.

Contents

Most numerical calculations of periodic turbulent boundary layers as well as pipe and channel flows have used turbulence models in a quasisteady manner, i.e., in the same form as used in the earlier steady-flow calculations, without rigorous justification.[1] This paper examines the implications of such "quasisteady" usage, especially in cases where the time scale of the externally imposed periodicity is comparable to the characteristic time scale of turbulent fluctuations in the flow.

Equations and Models

Considered is the fully developed turbulent flow in a long circular pipe of radius R in which the cross-sectional average velocity $\langle U_m \rangle$ (the notation $\langle\ \rangle$ representing ensemble average) is *forced* to vary periodically with time t according to

$$\langle U_m \rangle = \bar{U}_m \, (1+\gamma_{U_m} \cos 2\pi f_{os} t) \tag{1}$$

where γ_{U_m} and f_{os} are the amplitude and frequency of oscillation of $\langle U_m \rangle$ around the time mean value $\bar{U}_m$. The turbulence closure model chosen is the Prandtl energy model. The version chosen is identical to that used by Acharya and Reynolds[2] and is used in a quasisteady manner. Thus, the eddy viscosity $\langle \nu_t \rangle$ at radius r (distance y from the wall) is assumed to be equal to $C_1 \langle \nu_t' \rangle$, with the latter being related to the turbulent kinetic energy $\langle q^2 \rangle /2$ according to

$$\langle \nu_t' \rangle = \{ \langle q^2 \rangle \}^{1/2} \ell [1-\exp(-C_3 \langle q^2 \rangle^{1/2} y/\nu)] \tag{2}$$

where ν is the viscosity of the fluid, C_1 and C_3 are constants, and the turbulent length scale ℓ is prescribed as

$$\ell = \frac{4R}{30}\left[1-\left(\frac{r}{R}\right)^3\right] = \frac{4R}{30}\,[1-(1-\eta)^3] \tag{3}$$

The resulting model equations, after insertion of the turbulence closure model equation (2) and normalization with the time mean velocity $\bar{U}_m$ and pipe radius $D/2$, reduce to[1]

$$\xi \frac{\partial \langle U \rangle^*}{\partial \tau} = -\xi \frac{\partial \langle P_w \rangle^*}{\partial x} + \frac{\partial}{\partial \xi}\left[\xi\left(\frac{2}{Re}+C_1 \langle \nu_t' \rangle^*\right)\frac{\partial \langle U \rangle^*}{\partial \xi}\right] \tag{4}$$

$$\xi \frac{\partial \langle q^2 \rangle^*}{\partial \tau} = 2\xi \langle \nu_t \rangle^* \left(\frac{\partial \langle U \rangle^*}{\partial \xi}\right) - 2\xi \langle D \rangle^* + \frac{\partial}{\partial \xi}\left[\xi\left(\frac{2}{Re}+C_2 \langle \nu_t' \rangle^*\right)\frac{\partial \langle q^2 \rangle^*}{\partial \xi}\right] \tag{5}$$

where $\tau = \bar{U}_m t/R$, $\xi = y/R$, P_w is the wall static pressure, and Re the Reynolds number $\bar{U}_m D/\nu$. All the starred symbols denote normalized quantities. The dissipation term $\langle D \rangle^*$ in Eq. (5) is modeled as

$$\langle D \rangle^* = C_4 \frac{\langle q^2 \rangle^{*3/2}}{\ell^*}\left(1+\frac{2C_5}{\langle q^2 \rangle^{*1/2} \ell^* Re}\right) \tag{6}$$

The constants C_1, C_2, C_3, C_4, and C_5 are given the same values as used by Acharya and Reynolds,[2] namely: $C_1 = C_2 = 0.39$, $C_3 = 0.0136$, $C_4 = 0.0593$, and $C_5 = 2.698$. It is seen that the model chosen here is an "one-equation" model. This model is considered to be adequate to bring out the implications of quasisteady modeling for unsteady turbulent flows. The model equations (4) and (5) are solved numerically using a suitable finite difference procedure. The calculations are started with arbitrary initial conditions and continued until a purely periodic solution independent of the initial conditions is obtained. Two hypothetical periodic pipe flow situations were considered in which a suitable pressure gradient was imposed so as to give $\langle U_m \rangle$ according to Eq. (1). The two cases considered correspond to 1) Strouhal number $S_t = f_{os} R/\bar{U}_m = 0.07$, $\gamma_{U_m} \approx 0.4$, and $Re = 5\times 10^4$; and 2) $S_t = 0.0005$, $\gamma_{U_m} \approx 0.4$, and $Re = 5\times 10^4$. These Strouhal numbers can be compared with the value of about 0.1, corresponding to the (estimated) turbulent bursting frequency in the flow.[1] Thus, the first case can be regarded as "high frequency" and the second case can be regarded as "quasisteady."

Results and Comparison with Experiments

Figure 1a shows the predicted distribution of $\bar{U}$ in the inner layer coordinates, with u_* being the shear velocity. It is seen that the present model predicts almost identical mean velocity profiles at the two extreme frequencies. The quasisteady model used here thus implies that imposed oscillations have no effect on the time-averaged velocity profile, even at frequencies comparable to the turbulent bursting frequency. This conclusion has also been reached by others who used other turbulence models of less or more complexity. Note that any difference, observed in Fig. 1a, between the predicted velocity profiles for steady and unsteady flows is purely an "amplitude" effect, arising from the Reynolds number

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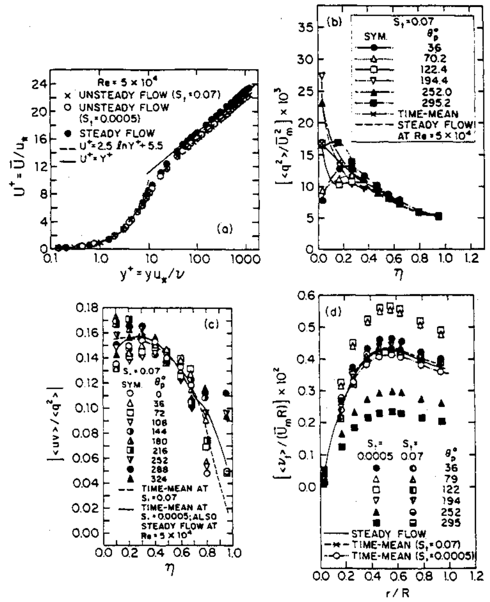

Fig. 1 Results of numerical calculations of periodic pipe flow.

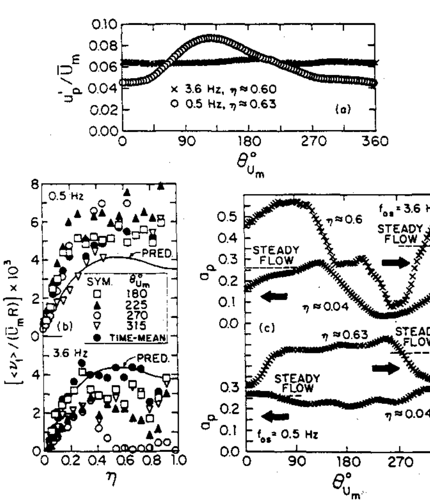

Fig. 2 Experimental results on periodic pipe flow.

dependence of the solution. Figure 1b shows the predicted distribution of $\langle q^2 \rangle$ at several typical phase angles θ_p (relative to $\langle dP_w/dx \rangle$ in the oscillation cycle for $S_t = 0.07$. At very low frequency ($S_t = 0.0005$), the unsteady flow can be expected to behave as a quasisteady flow. At the high frequency ($S_t = 0.07$), there is a strong interaction between the turbulent structure and the imposed oscillation. The figure shows that $\langle q^2 \rangle$ is "frozen" in time for $\eta > 0.6$ (or $\xi = r/R < 0.4$). The Reynolds shear stress $\langle uv \rangle$ was also found to exhibit qualitatively a similar behavior. However, quantitative effects were found to be different on the two properties. This is seen from Fig. 1c, which shows the distribution of the so-called structure parameter ($|\langle uv \rangle / \langle q^2 \rangle|$). At the low oscillation frequency, the model predicts that the structure parameter at a given point in the pipe remains very nearly constant throughout the oscillation cycle and is almost equal to the value in steady flow at the mean Reynolds number. (In fact, these differences are too small to be seen in the figure, indicating that structural equilibrium is maintained during the unsteady flow.) On the other hand, the model predicts that at $S_t = 0.07$, the structure parameter varies significantly during the cycle at all points in the flow, thereby indicating a breakdown of the structural equilibrium. Finally, Fig. 1d shows that at low frequency, the eddy viscosity $\langle \nu_t \rangle$ varies in time qualitatively as in quasisteady flow. At high frequency, however, the eddy viscosity remains practically frozen throughout the entire cycle, especially in the core region. The frozen distribution is not very different from that corresponding either to the periodic flow at the lower oscillation frequency or steady flow at the mean Reynolds number.

The above implications will now be compared with experimental observations. The experiments were conducted in fully developed periodic flow of water at $Re = 5 \times 10^4$ in a pipe of 5 cm diameter. Two experimental conditions were studied: 1) $f_{os} = 3.6$ Hz ($S_t = 0.1$), $\gamma_{U_m} = 0.15$; and 2) $f_{os} = 0.5$ Hz ($S_t = 0.013$), $\gamma_{U_m} = 0.64$. The first case represents "high"-frequency oscillation and closely corresponds to one of the hypothetical cases considered earlier. The other experiment can be considered to represent "intermediate"-frequency oscillation, since significant departures from quasisteady flow were observed in this case. The typical variation of the ensemble-averaged longitudinal turbulence intensity u_p' with the phase angle θ_{U_m} (now relative to $\langle U_m \rangle$) during the cycle is shown in Fig. 2a. It can be assumed that the behavior of $\langle q^2 \rangle$ would be qualitatively similar to that of u_p'. It is seen that, at the high oscillation frequency, u_p' is nearly frozen throughout the oscillation cycle at a value approximately equal to its time mean value, exactly as implied by quasisteady modeling (see Fig. 1b). At the intermediate frequency, however, u_p' varies during a part of the cycle but remains frozen over the remaining part at a value lower than its time mean value, thus behaving in a distinctly nonquasisteady manner. Quasisteady modeling [Eq. (2)] implies that the eddy viscosity $\langle \nu_t \rangle$ should also behave like u_p' since the length scale is constant with time. However, Fig. 2b shows that at high frequency, $\langle \nu_t \rangle$ varies widely during the oscillation cycle, whereas the quasisteady model [Eq. (2)] implies a frozen eddy viscosity. This limitation of quasisteady modeling can be seen even at the lower oscillation frequency of 0.5 Hz. The eddy viscosity varies widely during the interval $270 < \theta_{U_m} < 315$ deg, while u_p' is nearly frozen during this part of the cycle, as seen from Fig. 2a. Lastly, Fig. 2c shows that at 3.6 Hz, the quantity a_p, defined as $a_p = |\langle uv \rangle / u_p'^2|$ (and hence similar to the structure parameter of Fig. 1c), varies widely over the entire cycle near the wall as well as away from the wall. This indicates a complete breakdown of structural equilibrium in this flow. Some breakdown of equilibrium can be observed even at 0.5 Hz, even though it is not as widespread and complete as at the higher frequency.

Conclusions

The present study suggests that quasisteady, eddy viscosity-type turbulence models do not correctly describe the ensemble-averaged flow even at moderately large frequencies (frequencies of one order smaller than the turbulent bursting frequency in the flow). Partial to total breakdown of structural equilibrium may occur in these flows as the frequency of oscillation increases. Therefore, it is necessary to use closure models that are not inconsistent with this behavior.

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References

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